

Algebra	Group	Section	No.	الإسم:
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[1] Complete the following statements:

- (a) A square matrix A is called symmetric if.....
- (b) A square matrix A is called singular if.....
- (c) A square matrix A has inverse if.....

[2] If $A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \end{bmatrix}$.

- (a) Find if possible: $A + B$, $A.B$ and $B.A$
- (b) Find the eigenvalues and eigenvectors of A
- (c) Find the eigenvalues of $f(A) = A^3 - 2A$
- (d) Find the eigenvalues of $f(A) = 2^A$
- (e) Find A^n

Mid-Term Exam Algebra (12– 4 –2014) Prep-Year Model Answer

[1](a) A square matrix A is called symmetric if $A^t = A$.

(b) A square matrix A is called singular if $|A| = 0$.

(c) A square matrix A has inverse A^{-1} if $|A| \neq 0$.

----- (3 Marks)

[2] If $A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \end{bmatrix}$

(a) $A + B$ does not exist, $B.A$ does not exist. $A.B = \begin{bmatrix} 10 & 6 & 0 \\ -1 & 3 & 3 \end{bmatrix}$

----- (3 Marks)

(b) $|A - \lambda I| = \begin{vmatrix} 2-\lambda & 4 \\ 1 & -1-\lambda \end{vmatrix} = (2-\lambda)(-1-\lambda) - 4 = 0$

Then, $\lambda^2 - \lambda - 6 = 0$. Then, the eigenvalues: $\lambda_1 = 3$, $\lambda_2 = -2$.

From the equation, $\begin{bmatrix} 2-\lambda & 4 \\ 1 & -1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

For: $\lambda_1 = 3$, $\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then $-x + 4y = 0$, $x - 4y = 0$

Then $x = 4y = \text{any number except } 0$

Put $y = 1$, we get $x = 4$ and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

For: $\lambda_2 = -2$, $\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then $4x + 4y = 0$, $x + y = 0$

Then $y = -x = \text{any number except } 0$

Put $x = 1$, we get $y = -1$ and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

----- (5 Marks)

(c) The eigenvalues of $f(A) = A^3 - 2A$ are $f(3) = 27 - 6 = 21$, $f(-2) = -8 + 4 = -4$.

----- (2 Marks)

(d) The eigenvalues of $f(A) = 2^A$ are $f(3) = 2^3 = 8$, $f(-2) = 2^{-2} = \frac{1}{4}$.

----- (2 Marks)

(e) Since $T = \begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix}$. Then $T^{-1} = -\frac{1}{5} \begin{bmatrix} -1 & -1 \\ -1 & 4 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix}$

Since $T^{-1}.A.T = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$

Then $A^n = T \cdot \begin{bmatrix} 3^n & 0 \\ 0 & (-2)^n \end{bmatrix} \cdot T^{-1} = \begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3^n & 0 \\ 0 & (-2)^n \end{bmatrix} \cdot \frac{1}{5} \begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix}$
 $= \frac{1}{5} \begin{bmatrix} 4(3)^n + (-2)^n & 4(3)^n - 4(-2)^n \\ (3)^n - (-2)^n & (3)^n + 4(-2)^n \end{bmatrix}$

----- (5 Marks)

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